Exercise 1

Find $F'(x)$ for the following integrals:

$$F(x) = \int_0^x e^{-x^2t^2}\,dt$$

Solution

The Leibnitz rule states that if

$$F(x) = \int_{g(x)}^{h(x)} f(x,t)\,dt,$$

then

$$F'(x) = f(x,h(x))\frac{dh}{dx} - f(x,g(x))\frac{dg}{dx} + \int_{g(x)}^{h(x)} \frac{\partial f}{\partial t}\,dt,$$

provided that f and $\partial f/\partial t$ are continuous. In this exercise, $g(x) = 0$, $h(x) = x$, and $f(x,t) = e^{-x^2t^2}$. Applying the rule gives us

$$F'(x) = e^{-x^4} \cdot 1 - 1 \cdot 0 + \int_0^x \frac{\partial}{\partial x} e^{-x^2t^2}\,dt.$$

Therefore,

$$F'(x) = e^{-x^4} - 2x\int_0^x t^2 e^{-x^2t^2}\,dt.$$

[TYPO: The answer at the back of the book is missing t^2 in the integral (multiplying $-2x$).