

## Exercise 12

Differentiate both sides of the following equations:

$$\sinh x + \ln(\sin x) = \int_0^x (3 + x - t)u(t) dt, \quad 0 < x < \frac{\pi}{2}$$

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### Solution

Differentiating both sides of the equation with respect to  $x$  gives us

$$\cosh x + \cot x = 3u(x) \cdot 1 - (3 + x)u(0) \cdot 0 + \int_0^x \frac{\partial}{\partial x} (3 + x - t)u(t) dt,$$

where we used the Leibnitz rule to differentiate the integral. Therefore,

$$\cosh x + \cot x = 3u(x) + \int_0^x u(t) dt, \quad 0 < x < \frac{\pi}{2}.$$