Exercise 2

Find $F'(x)$ for the following integrals:

$$F(x) = \int_x^{x^2} \ln(1 + xt)\, dt$$

Solution

The Leibnitz rule states that if

$$F(x) = \int_{g(x)}^{h(x)} f(x, t)\, dt,$$

then

$$F'(x) = f(x, h(x))\frac{dh}{dx} - f(x, g(x))\frac{dg}{dx} + \int_{g(x)}^{h(x)} \frac{\partial f}{\partial t}\, dt,$$

provided that f and $\partial f/\partial t$ are continuous. In this exercise, $g(x) = x$, $h(x) = x^2$, and $f(x, t) = \ln(1 + xt)$. Applying the rule gives us

$$F'(x) = \ln(1 + x^3) \cdot 2x - \ln(1 + x^2) \cdot 1 + \int_x^{x^2} \frac{\partial}{\partial x} \ln(1 + xt)\, dt.$$

Therefore,

$$F'(x) = 2x \ln(1 + x^3) - \ln(1 + x^2) + \int_x^{x^2} \frac{t}{1 + xt}\, dt.$$