

## Exercise 6

Find the sum of the following infinite series:

$$x + \frac{n}{9}x + \frac{n^2}{81}x + \frac{n^3}{729}x + \cdots, \quad 0 < n < 9$$

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### Solution

Inspecting the series, we see that it is geometric. The first term is

$$a_1 = x,$$

and the common ratio is

$$r = \frac{n}{9} < 1.$$

Therefore, the sum of the series is

$$\begin{aligned} S &= \frac{a_1}{1 - r} \\ &= \frac{1}{1 - \frac{n}{9}}x \\ &= \frac{9}{9 - n}x. \end{aligned}$$