Exercise 7

Find the sum of the following infinite series:

$$\pi x + \frac{\pi}{2}x + \frac{\pi}{4}x + \frac{\pi}{8}x + \cdots$$

Solution

Inspecting the series, we see that it is geometric. The first term is

$$a_1 = \pi x,$$

and the common ratio is

$$r = \frac{1}{2}.$$

Therefore, the sum of the series is

$$\begin{aligned} S &= \frac{a_1}{1-r} \\ &= \frac{\pi}{\frac{1}{2}}x \\ &= 2\pi x. \end{aligned}$$