

Exercise 10

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$u(x, t) = x + t^3 + \frac{1}{2}t^2 - \frac{1}{2}t - \int_0^t \int_0^1 (\tau - \xi) d\xi d\tau$$

[**TYPO:** The integrand should be $(\tau - \xi)u(\xi, \tau)$.]

Solution

This is a Volterra-Fredholm integral equation because the unknown function u appears inside a double (mixed) integral. It is of the second kind because u appears both inside and outside the integrals. It's inhomogeneous because of the $x + t^3 + (1/2)t^2 - (1/2)t$. It's not singular since none of the limits of integration are infinite and the integrand does not become infinite in the intervals of integration.