

Exercise 6

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$u(x) = x + \frac{1}{6}x^3 - \int_0^x (x-t)u(t) dt$$

Solution

This is a Volterra integral equation because one of the limits of integration is not constant. It is of the second kind because the unknown function u appears both inside and outside the integral. It's inhomogeneous because of the $x + (1/6)x^3$. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.