

Exercise 9

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$u(x) = \frac{3}{2}x + \frac{1}{6}x^3 - \int_0^x (x-t)u(t) dt - \int_0^1 xu(t) dt$$

Solution

This is a Volterra-Fredholm integral equation because the unknown function u appears inside two (disjoint) integrals, a Volterra integral where one of the limits of integration is not constant and a Fredholm integral where the limits of integration are constant. The integral equation is of the second kind because u appears both inside and outside the integrals. It's inhomogeneous because of the $(3/2)x + (1/6)x^3$. It's not singular since none of the limits of integration are infinite and the integrands do not become infinite in the intervals of integration.