Exercise 13

In Exercises 13–16, show that the given function $u(x)$ is a solution of the corresponding Volterra integro-differential equation:

$$u'(x) = 2 + x + x^2 - \int_0^x u(t)\,dt,\ u(0) = 1,\ u(x) = 1 + 2x$$

Solution

Substitute the function in question on both sides of the integro-differential equation.

$$\begin{aligned}
\frac{d}{dx}(1+2x) &\stackrel{?}{=} 2 + x + x^2 - \int_0^x (1+2t)\,dt \\
2 &\stackrel{?}{=} 2 + x + x^2 - (t + t^2)\Big|_0^x \\
&\stackrel{?}{=} 2 + x + x^2 - (x + x^2) \\
&= 2
\end{aligned}$$

Therefore,

$$u(x) = 1 + 2x$$

is a solution of the Volterra integro-differential equation.